Mathematical and Metaphysical Explanations

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Abstract | This paper explores the connection between metaphysics and mathematics. The paper has two goals. The modest goal is to compare metaphysical and mathematical explanations, emphasizing their similarities. The ambitious goal is to single out a special subset of mathematical explanations, i.e. Lange's distinctive mathematical explanations, and argue that they are actually grounding explanations.

Keywords | Mathematical Explanations; Metaphysical Explanations; Grounding

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1 Introduction

It seems to be plausible that mathematical explanations (MA-explanations) in the empirical world are based on relations between two orders of facts: mathematical facts and empirical facts. The matter of which relations are best placed to do the explanatory work is, however, subject to controversy. Several accounts have been proposed to this end, but the dispute is still ongoing among philosophers (for instance, Baker (2012), Pincock (2014), and Baron (2016)). Regardless of which account turns out to be the best, mathematics' explanatory power can be hardly denied.

Conversely, the debate over the status of metaphysical explanations (ME-explanations) has been reappraised. Some philosophers argue that if any metaphysical notion can be explanatory, then *grounding* appears to be a promising candidate (e.g., Fine (2012), deRosset (2013), Dasgupta (2014), and Trogdon (2018)). The purpose of this paper is to shed light on the connection between MA-explanations and ME-explanations.

In section 2, I introduce the notion of grounding and provide two reasons for seeking a connection between metaphysical and mathematical explanations. In section 3, I spell out the role of grounding in metaphysical explanations by examining the formal properties of grounding and explanations. In section 4, I distinguish two ways in which mathematical explanations can be acausal, and claim that grounding claims and some mathematical explanations are acausal in the same sense. In section 5, I extend Lange's notion of 'explanations by confstraint' to grounding, and introduce the concept of distinctive metaphysical explanations. In section 6, I argue that distinctive mathematical explanations provide us with information about a portion of grounding network. I examine a case study from VSEPR theory to illustrate my view.

2 On Metaphysical Grounding and Mathematical Explanations

Grounding plays an essential role in metaphysics by encoding metaphysical priority or, in other words, what is fundamental. Metaphysical grounding is a philosophical concept that is expressed by instances of words such as 'because of,' 'depend on', 'in virtue of', etc. Grounding is so pervasive in the philosophical discourse that it can be found in many philosophical core sentences.

Consider a sentence such as 'non-empty sets depend on their members', or 'true propositions depend on truth-makers'. Those sentences can be understood

as pointing to a metaphysical notion of dependence.[1] Indeed, it has recently become customary to analyze those expressions in terms of grounding. But the properties of such a notion are notoriously open to dispute, so grounding is being subjected to numerous controversies over its nature.

To avoid some contentious issues, I will maintain that grounding is plural, by which I mean that there are various notions subsumed under the word 'grounding'. Incidentally, the fact that many different properties have been attributed to grounding may favor a pluralistic approach.[2] For the sake of argument, I will hold that 'grounding' is said in many ways; what all grounding relations, or predicates, have in common is that they aim to characterize non-causal fundamentality among facts or entities. Then I will single out the properties that makes grounding explanatory.

I consider two reasons for seeking a connection between grounding and MA-explanations. On the one hand, I want to represent the relation *in virtue of* which mathematical facts explain physical facts. Call this the 'dependency requirement.' By way of an example, the fact that we can never walk over each of Königsberg's bridges just once and end up where we started is explained in virtue of the fact that Königsberg's bridges do not form a Eulerian path.[3] That is a physical fact that is explained by a mathematical fact. On the other hand, I am interested in what sort of explanatory relation holds between the physical, contingent world and the mathematical, necessary realm. Call this the 'modal explanatory requirement.' I hold the view that a satisfactory narrative of the relationship between grounding and MA-explanations must take into account both requirements.

Let me clarify further the terminology I will use in this paper. In a MA-explanation of a physical, contingent phenomenon, the *explanans* is a mathematical fact, while the *explanandum* is a physical fact. For example, the fact that I was not able to untie a trefoil knot is explained by a topological fact, i.e. it is impossible to untie a trefoil knot in three dimensions without cutting it.

In a ME-explanation the *explanans* and the *explanandum* are connected by linking grounding facts with grounded facts. Consider a public event such as a conference. What makes it a conference rather than, say, a football match (Dasgupta 2014, 3)? An answer may explain that a conference is such *in virtue of* its

[1] Not every usage of these words expresses metaphysical grounding. For instance, some usages point to causal relations such as 'the door slammed because of the wind.' In the pages that follow I will only focus on the grounding usage.

[2] See McDaniel (2017) for a pluralistic concept of grounding.

[3] A Eulerian path is a trail in a graph which visits every edge exactly once. The Euler path theorem states that it is impossible to construct such a Eulerian path for any connected graph with more than two vertexes of odd degree.

attendees acting in certain ways rather than others. Roughly, an event is a conference if its participants behave in a particular manner, such as giving talks, listening to the speaker and so on. The explanation in question does not highlight what causes the event to occur; rather, it highlights what must be the case for an event to occur: what grounds what.[4]

I will discuss three central points: grounding is an explanatory notion; MA-explanations and ME-explanations are both acausal; there are cases where MA-explanations and ME-explanations are both explanations by constraints.

3 Grounding Is Explanatory

Some philosophers (e.g. J. Wilson (2014)) argue that grounding is not truly explanatory. This should not come as particularly surprising, as there are some who doubt that grounding is even intelligible (e.g. Hofweber (2009) and Daly (2012)).

I take that any explanation must address why–questions. Grounding explanations are no different in such respect: they address why–questions by making salient facts that are fundamental. Standing on a beach on a sunny day, I ask myself why the sand is hotter than the sea. An explanatory answer that makes salient what is fundamental can go something like this: the sand is hotter *by virtue of* having lower specific heat than that of water. Here is an example of a philosophical why–question that aims to highlight a fundamental fact: why does {Socrates} exist? And here is an answer: the fact that {Socrates} exists depends on the fact that Socrates exists. In other words, the existence of Socrates' singleton depends on the existence of Socrates the philosopher.

To see an analogy between grounding and explanations, I point out grounding has formal properties that philosophers tipically associate with explanations (Raven 2015). Grounding and explanations form strict partial orders on facts.[5] Explanations have the following properties:

- Explanations are irreflexive (x does not explain itself).
- Explanations are transitive (if x explains y, and y explains z, then x explains z).

[4] A caveat is in order here. I am not committing myself to the view that grounding is indispensable for ME-explanations; that is, I am not saying that grounding relations are the only ones that guarantee ME-explanations. See Kovacs (2017) for an argument against the indispensability of grounding in ME-explanations.

[5] A strict partial order is a binary relation R, such that R is irreflexive, asymmetric and transitive.

- Explanations do not work backwards (if x explains y, then y does not explain x).

Likewise, philosophers generally agree that grounding relations are strict partial order.

- Grounding is irreflexive (x does not ground itself).
- Grounding is transitive (if x grounds y, and y grounds z, then x grounds z)
- Grounding is asymmetric (if x grounds y, then y does not ground x.

Grounding theorists have also raised several counter-examples to each of those proprieties, but I will stick to the common view that grounding is a strict partial order on facts.[6]

Even if we concede that grounding and explanations are both strict partial orders, this is not sufficient for grounding to be considered explanatory. Indeed there are strict partial orders relations that are not explanatory. Consider the relation 'x is taller than y.' That is a strict partial order, but it is hardly explanatory on its own, without further properties or a relevant context of utterance.

I put forward two additional formal properties that explanations have: hyperintensionality and non–monotonicity.[7] Grounding is hyperintensional in the sense that, given two intensionally equivalent propositions, p and q, if p grounds q, then q cannot swap place with p *salva veritate*, i.e. without change in truth–value.[8] In the same vein, explanations are hyperintensional in the sense that two intensionally equivalent explanations for the same phenomenon cannot be substituted *salva veritate*.[9] Hyperintensionality is worth mentioning because it allows us to distinguish grounding from supposedly non-explanatory notions such as supervenience (see Horgan (1993)).

The failure of monotonicity also highlights the similarity between grounding and explanation. Deductively valid implications are monotonic in that if $P \rightarrow C$ is valid, so too is the implication from $P \wedge Q$ to C (for any Q). Explanations and

[6] Some of the most interesting counterexamples are collected in Trogdon (2013).

[7] See Raven (2013) and Dasgupta (2014).

[8] The fact that Socrates exists and the fact that {Socrates} exists are intensionally equivalent because any possible world in which the first obtains is a world in which the second obtains and vice versa. The fact that Socrates exists grounds the fact that {Socrates} exists. However, it is not the case that the fact that {Socrates} exists grounds the fact that Socrates exists.

[9] Give two explanations that are logically equivalent it is possible that a person may believe the one but not the other.

grounding are non–monotonic (Rosen 2010, 116) in the sense that if p explains (or grounds) q, it does not follow that $p \wedge r$ explains (or grounds) q (for any r).

The debate on what counts as an explanation is quite extensive. But given that grounding shares core formal properties with explanations, it makes sense to see how far that analogy can take us. Let us then explore what explanatory role grounding can perform.

4 Mathematical and Metaphysical Explanations Are Both Acausal

In this chapter there are a couple of theses I want to examine. Not only will I claim that metaphysical and MA-explanations are acausal, but I will also argue that they are acausal in the same sense. I first elaborate on MA-explanations and then evaluate in what sense MA-explanations are acausal.

Let us consider two ways in which MA-explanations can be acausal. On the one hand, according to the ontic view (e.g. Baker (2012) and Colyvan (2012)), MA-explanations appeal to facts involving abstract objects. That view stems from the debate on indispensability argument in the philosophy of mathematics. The argument goes like this: MA-explanations are acausal because they involve quantification over abstract objects that are indispensable to the explanatory power of scientific theories. On the other hand, according to the modal view, the *explanandum* is shown to hold necessarily regardless of contingent facts (Lange 2013). In the modal view, mathematics does not gain its explanatory strength from quantification over abstract entities, but it exploits facts that are modally stronger than ordinary causal laws.

The modal view is perhaps less known in the literature, so let me bring up an example (488). Why does a mother fail to distribute 23 strawberries evenly among her children? According to the modal view, an explanation of that fact involves two classes of facts: contingent facts and necessary facts. In our example, the contingent facts involve the mother's disposition, the number of strawberries, the number of children and so on. The necessary fact is a mathematical one, i.e. 23 cannot be divided evenly by 3. Even if the mother wanted to distribute the strawberries among her children, the mother could not do so without cutting the berries. The explanation works by showing that a necessary mathematical fact explains a contingent fact. 'Necessity' is key word in this context.

To me, a fascinating problem is whether we can connect Lange's modal view on mathematical explanations with grounding. Philosophers think that grounding can help us achieve two goals at least. First, grounding aims to set apart facts that are fundamental from facts that are merely derivative. Second, grounding aims

to get a grip on what *acausal* priority is. Consider the following Clark and Liggins (2012)'s example. Why a cup is brittle? The brittleness of the cup results from the specific way its constituent atoms are arranged. In other words, the atoms' configuration explains why the cup is brittle. The explanation is acausal because the arrangement of atoms does not, strictly speaking, cause the brittleness of the cup. In grounding jargon, the cup's brittleness is grounded in how its constituent atoms are arranged.

Clark and Liggins seem to admit that metaphysical explanations can exist in terms of grouding. Unfortunately, for my purpose, their example does not fit well with Lange's modal view. One could object that the atomic structure *causally* explains the brittleness of the cup by recognizing a physical factor, i.e. the lack of crystal structure, which is responsible for brittleness.

The problem is that the modal view leverages a broad notion of causality, in which an explanation is causal if it provides information about the world's causal network. This means that an explanation could still be causal even though it does not, strictly speaking, mention any causes. Lange (2016, xvii) himself points out that grounding explanations seem to describe features of the world's causal network. If Lange's claim is true, then I do not see an easy way to bridge the gap between the modal view and grounding.

5 Two Explanations by Constraints

Here is a plan to go beyond Lange. First, explain the concept of explanation by 'constraints'; second, show that ME-explanations are special kinds of explanation by constraints; third, detach grounding from causation by leveraging the concept of explanation by constraint.[10] That will pave the way for some ME-explanations to be acausal.

Mathematics can provide explanations by constraints in the sense that they can put modal constraints that are stronger than any physical law. For examples, Königsberg's bridges have never been crossed in a certain way because they *cannot* be crossed in that manner; I *cannot* untie the trefoil knot on my table because a trefoil knot is a nontrivial knot; the mother *cannot* distribute the strawberries among her children because 23 cannot be divided evenly by 3.

Those explanations seem to be distinctively mathematical, in the sense that the connection between the *explanans* and the *explanandum* holds not by any

[10] Some authors have recently advanced a strong connection between grounding and causation. See Schaffer (2016), A. Wilson (2018), and Trogdon (2018).

contingent law of nature but by mathematical facts. Mathematical facts are also necessary because they are true in every possible world.

Consider the following objection. Mathematics is not a unitary corpus, but there are many different mathematical theories. Some of them are also inconsistent with one another. This, however, introduces an element of contingency, which might be problematic for the present proposal.[11]

The objection seems to stem from a pluralistic view of mathematics. Does mathematical pluralism imply that mathematical facts are contingent? Let me clarify that a full-blown discussion on mathematical pluralism goes beyond the scope of this paper. In fact, there may be varieties of mathematical pluralism that are incompatible with my view. It is important to clarify that mathematical pluralism is not problematic as long as it does not hold that the relationship between the *explanans* and the *explanandum* is contingent by its nature. My view allows that there are cases in which the relationship between mathematical facts and physical facts is contingent. For example, there could be a physical fact that is explained by two mathematical theories that are incompatibe with one another. However, those explanations would not be explanations by constraints in the sense that I am interested in. In my lingo, an explanation by constraint must hold in virtue of a necessary mathematical relation. If it does not, the explanation could still be mathematical but not by constraint.

Distinctive mathematical explanations (DMA-explanations) diverge from explanations where mathematics is merely used to describe physical laws. To see what I mean, compare the aforementioned DMA-explanations with the following use of mathematics in Dalton's law. Why can climbers die of hypoxia climbing Mount Everest? An explanation involves Dalton's law of partial pressures, which states that the total pressure of a mixture of gas, P_T, is the sum of all of the partial pressures of the individual gases, $p_1 + p_2 + \ldots + p_n$, in the mixture. Persistent people who keep climbing Mount Everest experience the total pressure to decrease, causing the partial pressure of oxygen to decrease below 0.1 atm. Pressures of oxygen below 0.1 atm are not safe for humans, causing hypoxia.

Dalton's law explains hypoxia in climbers by leveraging a mathematical equation that represents the pressures of gas molecules as real numbers. However, the explanatory power of Dalton's law does not rely on necessary mathematical facts. The law explains by virtue of contingent aspects of the physical world, namely the fact that the total pressure in a mixture is constant.

DMA-explanations work differently by putting modal constraints on the physical world. My hypothesis is that DMA-explanations are not the only explanations

[11] I thank an anonymous referee for pointing out this objection.

by constraints. There are some ME-explanations that do not derive their power from contingent facts either; they do not hold by virtue of physical laws but by necessary grounding facts.

I distinguish between two classes of ME-explanations: distinctive metaphysical explanations (DME-explanations) and surreptitious metaphysical explanations. DME-explanations are *necessary* determination relations, whereas mere metaphysical explanations are *contingent* determination relations.

Surreptitious metaphysical explanations are quite similar to causal explanations in that they works as surrogates of causation. In Dalton's law the total pressure of a gas mixture is grounded in each partial pressure. As a result, each partial pressure is fundamental, whereas the total pressure is merely derivative. Since the explanation in question works by appealing to contingent aspects of the world, i.e. a law of nature, it is a surreptitious metaphysical explanation. In my view, 'Clark & Liggins' example mentioned earlier is also surreptitious, because it gains its explanatory power from a contingent aspect of the world, i.e. the lack of crystal structure.

The distinction between surreptitious and DME-explanations may cast light on why some philosophers attribute the property of necessity to grounding, whereas others ascribe to it the property of contingency.[12] In this paper, I will not elaborate further on surreptitious metaphysical explanations because they are not explanations by constraints.

DME-explanations have strong modal force. In any DME-explanation, the *explanans* necessarily entails the *explanandum*. Although there is more to grounding than modality (Fine 2012), there is a strong connection between grounding and necessitation. Gievn a plurality of facts, F, F is a DME-explanation of a plurality of facts, P, iff P is fully grounded in F, and P is modally entailed by F.[13] In plain words, DME-explanations convey metaphysical necessity.

I argue that DME-explanations are explanations by constraints. They do not work by describing the world's causal network, but they derive their power from necessary (grounding) facts. The fact that Socrates exists is a DME-explanation of the fact that {Socrates} exists. This is because the existence of Socrates fully grounds the existence of {Socrates}, and the existence of {Socrates} is modally entailed by the existence of Socrates because every world where Socrates exists is a world where {Socrates} exists too.

Consider the following objection. The existence of Socrates is a contingent.

[12] Necessitantism is the default view about grounding. See Rosen (2010), deRosset (2010), Audi (2012), Trogdon (2013), and Dasgupta (2014). For contingentism see Leuenberger (n.d.) and Skiles (2015).

[13] It is common to distinguish full and partial grounding. For example, A and B together fully ground $A \wedge B$, while each only partially grounds it.

Thus, the power of a DME-explanation, such as the one that explains the existence of {Socrates}, seems to rely on a contingent fact. If that is true, how can a DME-explanation be an explanation by constraint? After all, it does not seem to carry any modal necessity.

My reply to that objection goes as follows. Suppose someone had killed Socrates' father before Socrates was born, so that Socrates would never have existed. If that had occurred, Socrates' existence would not have explained the existence of its singleton. However, the success of a DME-explanation does not presuppose that Socrates and its singleton exist in every world. but that every world where Socrates exists is a world where {Socrates} exists too. To put it another way, the necessary grounding fact is the connection between Socrates and its singleton, not the existence of Socrates.[14]

6 Distinctive Mathematical Explanations and Grounding Network

I argued that DMA-explanations and DME-explanations are explanations by constraints. I now want to establish a stronger connection between the two. In a nutshell, here is my hypothesis: if a mathematical fact distinctively explains an empirical fact, then the former fully grounds the latter. DMA-explanations are a subset of DME-explanation or, in other words, DMA-explanations are DME-explanations in which the *explanandum* is mathematical in character. If that is true, then DMA-explanations can provide us with information about a portion of the (acausal) grounding network.

In a DMA-explanation, mathematical truths modally entail a plurality of physical phenomena, in the sense that DMA-explanations uniquely derive their power from necessary mathematical truths. I define a mathematical necessity as a mathematical truth that is metaphysically necessary. The goal is to show that some mathematical truths ground physical phenomena. A DMA-explanation contains a necessary mathematical truth, which entails the *explanandum* in every possible world.

As a consequence, we should regard mathematical facts as metaphysically fundamental or, in other words, as necessary grounding facts. A DMA-explanation contains a specific necessary grounding fact, i.e. a mathematical truth, which entails the *explanandum* in every possible world. Basically, DMA-explanations mirror ME-explanations that exploit mathematical truths as necessary grounding facts.

[14] I do not believe that this move can be applied to Clark and Liggins (2012)'s example. It does not seem that every world where a cup is brittle is a world where its atoms are arranged in a certain way. There could be worlds in which brittleness is caused by other atoms' arrangements.

It may be easier to see what I mean by way of an example. Consider again the bridges of Königsberg. No one can cross the bridges of Königsberg by passing each bridge exactly once and then going back to the starting point. This is explained by a mathematical fact: Königsberg's bridges form a non-Eulerian graph. The explanatory power does not depend upon any causal phenomenon; rather, it appeals to a mathematical fact: necessarily, such-and-such an arrangement is non-Eulerian.

The connection between the *explanans* and the *explandum* holds necessarily given the fixed arrangement of bridges: every possible world in which a sequence of bridges does not form a Eulerian path is a world where no one can cross Königsberg's bridges. Also, mathematical facts are (metaphysically) necessarly true in every possible world, and that metaphysical necessities are grounding facts. The *explanans* is a grounding fact that explains by its necessary modal force given by a mathematical result. Because the connection between the grounding fact and the grounded fact is necessary, the *explanans* fully grounds the *explanandum*.

The case of Königsberg's bridges shows that a DMA-explanation mirrors a corresponding DME-explanation. But let me show you another example that has not been studied in the literature so far.

The shape of a molecule depends on the minimum repulsion energy of the electron groups. The bond angles within a molecule can be determined experimentally, but it is possible to make predictions *ab initio* based on pure mathematical considerations. VSEPR theory allows us to predict the shape of molecules given the assumption that the electron groups want to get as far apart as possible.[15] That is a consequence of the Pauli principle combined with electrostatic repulsion. The Pauli exclusion principle states that no more than two electrons may fill the same orbital, and that if two electrons are present they must have opposite spins. Moreover, the electron groups are attracted to the nucleus, but they also repel one another through coulombic forces.

Consider now one molecule of CH_4 (methane) with four hydrogen atoms around the central carbon atom. In the absence of distortions, the four hydrogen atoms achieve the maximum separation possible by assuming the regular tetrahedral shape, where the hydrogen atoms are the vertexes of the tetrahedron. That is a mathematical necessary constraint, because it is the only way for the hydrogen atoms to be equidistant from one another and the carbon atom in a three-dimensional space.[16]

The explanation of the molecular arrangement of CH_4 gets its power from a

[15] I will follow Gillespie et al. (2001, 79–85) in the discussion of this phenomenon.

[16] See Glaister (1993) for a concise discussion of the tetrahedral shape of methane.

mathematical necessary fact: the central angle between *any* two vertexes of a regular tetrahedron is $\arccos(-\frac{1}{3}) \approx 109.47^{\circ}$. Note that the explanation appeals to contingent facts no more than the case of Königsberg's bridges presupposes a fixed arrangement of bridges. Thus, it is a DMA-explanation. Because the explanation works by exploiting a necessary mathematical truths, we can say that the molecular arrangement of CH_4 is fully grounded in the regular tetrahedron having certain mathematical properties. The DMA-explanation mirrors a corresponding DME-explanation.

7 Conclusion

My argument shows that DMA-explanations are of great value for metaphysics. To sum up, I argued that there is a strong connection between DMA-explanations and DME-explanations. They both appeal to facts that are modally stronger than ordinary causal facts. In any DMA-explanation, the connection between the *explanandum* and the *explanans* holds by virtue of a necessary mathematical fact underlying the explanation. This fact also figures in a corresponding DME-explanation, in that it fully grounds a plurality of physical phenomena. My claim is that we can track what facts are fully grounded in mathematical truths by identifying DMA-explanations.

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